

# CSCI 2244 – Homework 9

Out: **Saturday**, November 16, 2019

Due: **Saturday**, November 23, 2019, 11:59pm

This homework consists of written exercises. You *must* type your solutions. See the “Assignments” section in the syllabus for advice about doing this. You should submit your homework via Canvas. In particular, you should upload a pdf file called:

`FirstName_LastName_Homework9.pdf`

Please use your full first name and last name, as they appear in official university records. The reason for doing so is that the TAs and I must match up these names with the entries in the gradebook.

## 1 Martingales

**Task 1.1** (10 pts). You flip a fair coin repeatedly until the sequence TTHT occurs. Let  $N$  be the total number of coin flips you make. Find  $\mathbb{E}[N]$  by **two methods**: first using Markov chain techniques, and then by constructing an appropriate martingale and using the stopping theorem.

**Task 1.2** (5 pts). You roll a fair 6-sided die. Let  $N$  be the value shown. You then flip  $N$  fair coins. Let  $H$  be the number of heads you obtain. What is  $\mathbb{E}[H]$ ?

**Task 1.3** (10 pts). You repeatedly roll a fair 6-sided die until you roll three 1s in a row. Let  $X$  be the sum of all your die roll values. (For example, if your sequence of rolls was 2, 6, 1, 1, 1, then  $X$  would be 11). What is  $\mathbb{E}[X]$ ?

**Task 1.4** (5 pts). Let  $X_1, X_2, \dots$  be a sequence of independent and identically distributed random variables. Let  $\mathbb{E}[X_1] = 0$  and  $\text{Var}[X_1] = \sigma^2$ . Set

$$Z_n = \left( \sum_{i=1}^n X_i \right)^2 - n\sigma^2$$

Show that  $Z_1, Z_2, \dots$  is a martingale.